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CONCENTRIC MAGIC CUBES OF PRIME NUMBERS

Abstract

Like their two-dimensional counterparts, three-dimensional magic cubes can fascinate and surprise you with their existence. The level of difficulty and fascination is even higher when magic cubes are constructed inside magic cubes. When there are several layers, they remind you of the Russian Matryoshka dolls-within-dolls. Several examples of magic cubes constructed using formulas are given in this paper, including some constructed from prime numbers.

Introduction

On the web page

http://www.magic-SquareS.net/c-t-htm/c_prime.htm

there is the concentric magic cube of order 6 of distinct primes seen in figure 1.

Figure 1

4831	4783	67	9811	4639	5479
191	241	193	9473	9769	9743
331	577	5009	4751	9619	9323
8273	9719	8933	1123	829	733
8423	7499	8287	1789	1801	1811
7561	6791	7121	2663	2953	2521

131	761	379	9403	9497	9439
8951	2437	3547	5309	8447	919
9643	3209	5573	2281	8677	227
2143	8243	4877	6007	613	7727
8311	5851	5743	6143	2003	1559
431	9109	9491	467	373	9739

337	8849	8821	1409	1307	8887
7013	5903	2879	9007	1951	2857
8009	3217	2767	8117	5639	1861
9049	6073	5521	2333	5813	821
4219	4547	8573	283	6337	5651
983	1021	1049	8461	8563	9533

8543	8839	9277	173	1831	947
4177	3533	9587	1297	5323	5693
7487	4057	7537	4349	3797	2383
31	4231	1753	7103	6653	9839
449	7919	863	6991	3967	9421
8923	1031	593	9697	8039	1327

8419	3299	8317	1607	5419	2549
9151	7867	3727	4127	4019	719
3593	9257	3863	4993	1627	6277
977	1193	7589	4297	6661	8893
149	1423	4561	6323	7433	9721
7321	6571	1553	8263	4451	1451

7349	3079	2749	7207	6917	2309
127	9629	9677	397	101	9679
547	9293	4861	5119	251	9539
9137	151	937	8747	9041	1597
8059	2371	1583	8081	8069	1447
4391	5087	9803	59	5231	5039

The magic constant of this magic cube is 29610.

Inside is the associative (central symmetric) and pantriagonal cube of order 4 with a magic constant of 19740 that is seen in figure 2.

Figure 2

2437	3547	5309	8447
3209	5573	2281	8677
8243	4877	6007	613
5851	5743	6143	2003

5903	2879	9007	1951
3217	2767	8117	5639
6073	5521	2333	5813
4547	8573	283	6337

3533	9587	1297	5323
4057	7537	4349	3797
4231	1753	7103	6653
7919	863	6991	3967

7867	3727	4127	4019
9257	3863	4993	1627
1193	7589	4297	6661
1423	4561	6323	7433

The concentric magic cube is an interesting subset of the set of magic cubes.

Our next step will be to present concentric magic cubes of orders 5, 6 and 7 that are composed of distinct primes.

Concentric magic cubes of order 5

The scheme for concentric magic cubes of order 5 is as seen in figure 3

Figure 3

x1	x2	x3	x4	x5
x6	x7	x8	x9	x10
x11	x12	x13	x14	x15
x16	x17	x18	x19	x20
x21	x22	x23	x24	x25

x26	x27	x28	x29	x30
x31				k-x31
x32				k-x32
x33				k-x33
k-x30	k-x27	k-x28	k-x29	k-x26

x34	x35	x36	x37	x38
x39				k-x39
x40		k/2		k-x40
x41				k-x41
k-x38	k-x35	k-x36	k-x37	k-x34

x42	x43	x44	x45	x46
x47				k-x47
x48				k-x48
x49				k-x49
k-x46	k-x43	k-x44	k-x45	k-x42

k-x25	k-x22	k-x23	k-x24	k-x21
k-x10	k-x7	k-x8	k-x9	k-x6
k-x15	k-x12	k-x13	k-x14	k-x11
k-x20	k-x17	k-x18	k-x19	k-x16
k-x5	k-x2	k-x3	k-x4	k-x1

The magic constant of this magic cube S = 5k / 2.

In the inner region we put the magic cube of order 3 with magic constant S = 3k / 2. This magic cube is associative with constant associativity k.

We make this magic cube in the first stage.

If such a magic cube of order 3 is found, the second stage is the working out of the edging.

The system of linear equations describing the concentric cube of order 5 follows.

x1+x2+x3+x4+x5=S
x6+x7+x8+x9+x10=S
x11+x12+x13+x14+x15=S
x16+x17+x18+x19+x20=S
x21+x22+x23+x24+x25=S
x26+x27+x28+x29+x30=S
x34+x35+x36+x37+x38=S
x42+x43+x44+x45+x46=S

x1+x6+x11+x16+x21=S
x2+x7+x12+x17+x22=S
x3+x8+x13+x18+x23=S
x4+x9+x14+x19+x24=S
x5+x10+x15+x20+x25=S
x26+x31+x32+x33-x30=3S/5
x34+x39+x40+x41-x38=3S/5
x42+x47+x48+x49-x46=3S/5
x1+x26+x34+x42-x25=3S/5
x2+x27+x35+x43-x22=3S/5
x3+x28+x36+x44-x23=3S/5
x4+x29+x37+x45-x24=3S/5
x5+x30+x38+x46-x21=3S/5
x6+x31+x39+x47-x10=3S/5
x11+x32+x40+x48-x15=3S/5
x16+x33+x41+x49-x20=3S/5

This system is solved to find the general formula for the concentric magic cube of order 5.

x5=S-x1-x2-x3-x4
x9=S-x10-x6-x7-x8
x15=S-x11-x12-x13-x14
x17=S-x16-x18-x19-x20
x21=S-x1-x11-x16-x6
x22=-x12+x16+x18+x19-x2+x20-x7
x23=S-x13-x18-x3-x8
x24=x10-x14-x19-x4+x6+x7+x8
x25=-S+x1-x10+x11+x12+x13+x14+x2-x20+x3+x4
x30=S-x26-x27-x28-x29
x33=(8 S)/5-2 x26-x27-x28-x29-x31-x32
x34=-S-x10+x11+x12+x13+x14+x2-x20-x26+x3+x4-x46+x47+x48+x49
x35=S/5-x12+x16+x18+x19-2 x2+x20-x27+x44+x45+2 x46-x47-x48-x49-x7
x36=(8 S)/5-x13-x18-x28-2 x3-x44-x8
x37=(3 S)/5+x10-x14-x19-x29-2 x4-x45+x6+x7+x8
x38=-((2 S)/5)-x11-x16+x2+x26+x27+x28+x29+x3+x4-x46-x6
x39=(3 S)/5+x10-x31-x47-x6
x40=(8 S)/5-2 x11-x12-x13-x14-x32-x48
x41=-S-x16+x20+2 x26+x27+x28+x29+x31+x32-x49
x42=(3 S)/5+x46-x47-x48-x49
x43=(2 S)/5-x44-x45-2 x46+x47+x48+x49

This system as 28 of the 49 variables free once you set the parameter k.

Using this general formula, you can construct a lot of concentric magic cubes of order 5.
Here are some of my solutions for specific values of S and k.

Figure 4

S = 12955, k = 5182

4253	953	1301	2789	3659
701	2909	4133	4721	491
431	2099	2633	5051	2741
3989	3371	809	83	4703
3581	3623	4079	311	1361

29	2063	4799	3923	2141
2213	2939	4733	101	2969
4001	3863	659	3251	1181
3671	971	2381	4421	1511
3041	3119	383	1259	5153

173	3449	4271	599	4463
5171	4073	239	3461	11
4919	1979	2591	3203	263
1973	1721	4943	1109	3209
719	1733	911	4583	5009

4679	4931	1481	773	1091
179	761	2801	4211	5003
1163	1931	4523	1319	4019
2843	5081	449	2243	2339
4091	251	3701	4409	503

3821	1559	1103	4871	1601
4691	2273	1049	461	4481
2441	3083	2549	131	4751
479	1811	4373	5099	1193
1523	4229	3881	2393	929

Figure 5

S = 13945, k = 5578

2111	2039	5399	107	4289
5519	1187	2741	4271	227
557	4481	389	3221	5297
4817	1229	269	4139	3491
941	5009	5147	2207	641

1559	5507	1997	4877	5
857	2411	4259	1697	4721
5477	2957	2441	2969	101
479	2999	1667	3701	5099
5573	71	3581	701	4019

1571	4649	1907	4931	887
1901	4079	197	4091	3677
4751	2801	2789	2777	827
1031	1487	5381	1499	4547
4691	929	3671	647	4007

3767	1181	4211	659	4127
317	1877	3911	2579	5261
2879	2609	3137	2621	2699
5531	3881	1319	3167	47
1451	4397	1367	4919	1811

4937	569	431	3371	4637
5351	4391	2837	1307	59
281	1097	5189	2357	5021
2087	4349	5309	1439	761
1289	3539	179	5471	3467

Figure 6

S = 15835, k = 6334

3923	4337	941	5273	1361
431	6329	3533	71	5471
5693	1031	4457	4463	191
5507	4127	2633	857	2711
281	11	4271	5171	6101

683	4091	3557	5591	1913
5237	3491	5657	353	1097
947	3593	761	5147	5387
4547	2417	3083	4001	1787
4421	2243	2777	743	5651

5153	467	2963	3671	3581
5051	3677	593	5231	1283
2531	4721	3167	1613	3803
347	1103	5741	2657	5987
2753	5867	3371	2663	1181

5843	617	6311	137	2927
4253	2333	3251	3917	2081
521	1187	5573	2741	5813
1811	5981	677	2843	4523
3407	5717	23	6197	491

233	6323	2063	1163	6053
863	5	2801	6263	5903
6143	5303	1877	1871	641
3623	2207	3701	5477	827
4973	1997	5393	1061	2411

Figure 7

S = 15955, k = 6382

4001	5099	1439	3023	2393
479	6143	1901	1103	6329
5471	641	3923	5639	281
5981	353	2579	4889	2153
23	3719	6113	1301	4799

83	1709	6131	5189	2843
5441	3371	5153	1049	941
2243	3413	1889	4271	4139
4649	2789	2531	4253	1733
3539	4673	251	1193	6299

4409	4973	3833	2141	599
3671	4073	569	4931	2711
1109	4049	3191	2333	5273
983	1451	5813	2309	5399
5783	1409	2549	4241	1973

5879	1511	4283	521	3761
6311	2129	3851	3593	71
1031	2111	4493	2969	5351
113	5333	1229	3011	6269
2621	4871	2099	5861	503

1583	2663	269	5081	6359
53	239	4481	5279	5903
6101	5741	2459	743	911
4229	6029	3803	1493	401
3989	1283	4943	3359	2381

Figure 8

S = 18035, k = 7214

4297	5647	3571	157	4363
2707	967	2617	6841	4903
2887	4933	5791	2953	1471
1867	2971	3673	6793	2731
6277	3517	2383	1291	4567

523	6427	877	5197	5011
6481	3943	6571	307	733
5437	4657	1063	5101	1777
3391	2221	3187	5413	3823
2203	787	6337	2017	6691

4447	331	6163	541	6553
5683	5077	223	5521	1531
607	4051	3607	3163	6607
6637	1693	6991	2137	577
661	6883	1051	6673	2767

6121	1933	2593	6217	1171
853	1801	4027	4993	6361
3361	2113	6151	2557	3853
1657	6907	643	3271	5557
6043	5281	4621	997	1093

2647	3697	4831	5923	937
2311	6247	4597	373	4507
5743	2281	1423	4261	4327
4483	4243	3541	421	5347
2851	1567	3643	7057	2917

In figures 25, 27 and 28 you can see interior concentric magic cubes of order 5 inside concentric magic cubes of order 7.

It is interesting to note that we can create classic (numbers 1 though n^2) concentric magic cubes of order 5 (see Fig. 9). This magic cube is made up of distinct positive integers from 1 to 125.

Figure 9

S = 315, k = 126

16	98	62	25	114
44	75	112	50	34
43	74	49	113	36
115	41	73	48	38
97	27	19	79	93

21	10	70	125	89
94	81	106	2	32
91	100	22	67	35
72	8	61	120	54
37	116	56	1	105

122	68	5	103	17
46	102	18	69	80
7	30	63	96	119
31	57	108	24	95
109	58	121	23	4

123	40	71	15	66
39	6	65	118	87
84	59	104	26	42
9	124	20	45	117
60	86	55	111	3

33	99	107	47	29
92	51	14	76	82
90	52	77	13	83
88	85	53	78	11
12	28	64	101	110

Concentric magic cubes of order 6

A concentric magic cube of order 6 has already appeared in figure 1. A scheme for concentric magic cubes of order 6 is given in figure 10.

Figure 10

x1	x2	x3	x4	x5	x6
x7	x8	x9	x10	x11	x12
x13	x14	x15	x16	x17	x18
x19	x20	x21	x22	x23	x24
x25	x26	x27	x28	x29	x30
x31	x32	x33	x34	x35	x36

x37	x38	x39	x40	x41	x42
x43					k-x43
x44					k-x44
x45					k-x45
x46					k-x46
k-x42	k-x38	k-x39	k-x40	k-x41	k-x37

x47	x48	x49	x50	x51	x52
x53					k-x53
x54					k-x54
x55					k-x55
x56					k-x56
k-x52	k-x48	k-x49	k-x50	k-x51	k-x47

x57	x58	x59	x60	x61	x62
x63					k-x63
x64					k-x64
x65					k-x65
x66					k-x66
k-x62	k-x58	k-x59	k-x60	k-x61	k-x57

x67	x68	x69	x70	x71	x72
x73					k-x73
x74					k-x74
x75					k-x75
x76					k-x76
k-x72	k-x68	k-x69	k-x70	k-x71	k-x67

k-x36	k-x32	k-x33	k-x34	k-x35	k-x31
k-x12	k-x8	k-x9	k-x10	k-x11	k-x7
k-x18	k-x14	k-x15	k-x16	k-x17	k-x13
k-x24	k-x20	k-x21	k-x22	k-x23	k-x19
k-x30	k-x26	k-x27	k-x28	k-x29	k-x25

The magic constant of the cube S = 3k. Inside there is a magic cube of order 4 with a magic constant S = 2k, this magic cube can be both associative and non-associative. In magic cube of figure 1 is an example of an interior magic cube that is associative.

In the present scheme can write a system of linear equations and solve it. The end result is a general formula for concentric magic cubes of order 6, similar to that shown for the concentric magic cube of order 5.
Using distinct primes I was able to create several concentric magic cubes of order 6 with a magic constant S < 29610. Here are some examples.

Figure 11

S = 5670, k = 1890

971	761	1801	367	157	1613
1447	379	491	1031	569	1753
1033	1667	1709	281	877	103
1117	1777	419	457	1607	293
461	23	787	1723	797	1879
641	1063	463	1811	1663	29

607	409	719	1877	1627	431
1543	1039	31	887	1823	347
599	439	2311	383	647	1291
233	353	557	2371	499	1657
1229	1949	881	139	811	661
1459	1481	1171	13	263	1283

937	149	1303	677	983	1621
1553	97	307	2069	1307	337
727	2267	523	317	673	1163
653	967	1451	271	1091	1237
1531	449	1499	1123	709	359
269	1741	587	1213	907	953

1277	1693	19	1289	829	563
593	53	2333	751	643	1297
1423	991	107	373	2309	467
311	2423	571	659	127	1579
739	313	769	1997	701	1151
1327	197	1871	601	1061	613

17	1831	401	1381	1847	193
397	2591	1109	73	7	1493
101	83	839	2707	151	1789
1759	37	1201	479	2063	131
1699	1069	631	521	1559	191
1697	59	1489	509	43	1873

1861	827	1427	79	227	1249
137	1511	1399	859	1321	443
1787	223	181	1609	1013	857
1597	113	1471	1433	283	773
11	1867	1103	167	1093	1429
277	1129	89	1523	1733	919

This magic cube is a known minimal simple magic cube.

Figure 12

S = 6030, k = 2010

13	59	1319	1439	1987	1213
1811	479	397	257	1759	1327
1973	383	839	1277	1091	467
1103	1949	1283	461	353	881
991	1259	499	1409	439	1433
139	1901	1693	1187	401	709

7	389	1777	1487	1913	457
1033	1861	19	641	1499	977
643	877	1597	1373	173	1367
1847	1193	71	1459	1297	163
947	89	2333	547	1051	1063
1553	1621	233	523	97	2003

809	1601	1291	557	31	1741
1567	151	73	1667	2129	443
1151	953	1723	587	757	859
787	283	1733	1303	701	1223
1447	2633	491	463	433	563
269	409	719	1453	1979	1201

1993	1999	1013	487	311	227
79	449	1889	1471	211	1931
347	1279	107	2017	617	1663
941	1061	1867	431	661	1069
887	1231	157	101	2531	1123
1783	11	997	1523	1699	17

1907	1873	313	1237	179	521
857	1559	2039	241	181	1153
373	911	593	43	2473	1637
223	1483	349	827	1361	1787
1181	67	1039	2909	5	829
1489	137	1697	773	1831	103

1301	109	317	823	1609	1871
683	1531	1613	1753	251	199
1543	1627	1171	733	919	37
1129	61	727	1549	1657	907
577	751	1511	601	1571	1019
797	1951	691	571	23	1997

Figure 13

S = 10080, k = 3360

421	2347	2081	2971	1583	677
1277	1229	1307	3323	1571	1373
1549	1933	2477	293	1297	2531
2539	359	2069	2423	1951	739
2393	1861	653	499	2297	2377
1901	2351	1493	571	1381	2383

1481	41	197	2719	2521	3121
673	919	3331	1913	557	2687
1873	1663	151	2273	2633	1487
2851	3329	17	193	3181	509
2963	809	3221	2341	349	397
239	3319	3163	641	839	1879

1223	463	3079	857	2207	2251
2837	661	2269	3347	443	523
2903	3251	103	53	3313	457
647	2617	2309	61	1733	2713
1361	191	2039	3259	1231	1999
1109	2897	281	2503	1153	2137

2677	3083	2767	311	541	701
1693	2129	101	1321	3169	1667
1367	1627	3299	1051	743	1993
1181	47	3307	3257	109	2179
503	2917	13	1091	2699	2857
2659	277	593	3049	2819	683

3301	3137	89	433	1249	1871
1613	3011	1019	139	2551	1747
1559	179	3167	3343	31	1801
241	727	1087	3209	1697	3119
1877	2803	1447	29	2441	1483
1489	223	3271	2927	2111	59

977	1009	1867	2789	1979	1459
1987	2131	2053	37	1789	2083
829	1427	883	3067	2063	1811
2621	3001	1291	937	1409	821
983	1499	2707	2861	1063	967
2683	1013	1279	389	1777	2939

Figure 14

S = 19800, k = 6600

883	5591	3697	4243	233	5153
1117	4457	5923	1031	4799	2473
5387	4289	977	2273	5507	1367
4447	179	2861	6121	743	5449
6269	863	5323	1549	2777	3019
1697	4421	1019	4583	5741	2339

1783	47	239	6373	6007	5351
151	613	6571	4349	1667	6449
6247	3517	283	5147	4253	353
6211	6359	23	271	6547	389
4159	2711	6323	3433	733	2441
1249	6553	6361	227	593	4817

397	211	3299	5861	5981	4051
4597	1321	3391	6491	1997	2003
1523	6563	19	149	6469	5077
6221	4363	5273	31	3533	379
4513	953	4517	6529	1201	2087
2549	6389	3301	739	619	6203

6133	6113	3797	449	3229	79
3881	5399	71	2083	5647	2719
911	3067	6569	1327	2237	5689
1597	131	6451	6581	37	5003
757	4603	109	3209	5279	5843
6521	487	2803	6151	3371	467

6343	5659	3187	857	3491	263
5927	5867	3167	277	3889	673
499	53	6329	6577	241	6101
173	2347	1453	6317	3083	6427
521	4933	2251	29	5987	6079
6337	941	3413	5743	3109	257

4261	2179	5581	2017	859	4903
4127	2143	677	5569	1801	5483
5233	2311	5623	4327	1093	1213
1151	6421	3739	479	5857	2153
3581	5737	1277	5051	3823	331
1447	1009	2903	2357	6367	5717

Figure 15

S = 25200, k = 8400

5197	5521	5171	4373	677	4261
2311	2699	7237	1361	6029	5563
6803	919	4787	8053	311	4327
5087	7937	1543	1747	7433	1453
571	2267	3413	3469	8171	7309
5231	5857	3049	6197	2579	2287

523	809	11	8269	8011	7577
37	1213	8293	2621	4673	8363
8161	7573	643	5807	2777	239
8287	6911	137	2011	7741	113
7369	1103	7727	6361	1609	1031
823	7591	8389	131	389	7877

179	6763	5717	3389	5689	3463
8317	709	6397	6287	3407	83
2503	8387	1399	2153	4861	5897
2143	727	7823	31	8219	6257
7121	6977	1181	8329	313	1279
4937	1637	2683	5011	2711	8221

4951	7537	1667	1567	4241	5237
7901	8087	71	7219	1423	499
1187	181	8369	577	7673	7213
1297	3539	6247	7001	13	7103
6701	4993	2113	2003	7691	1699
3163	863	6733	6833	4159	3449

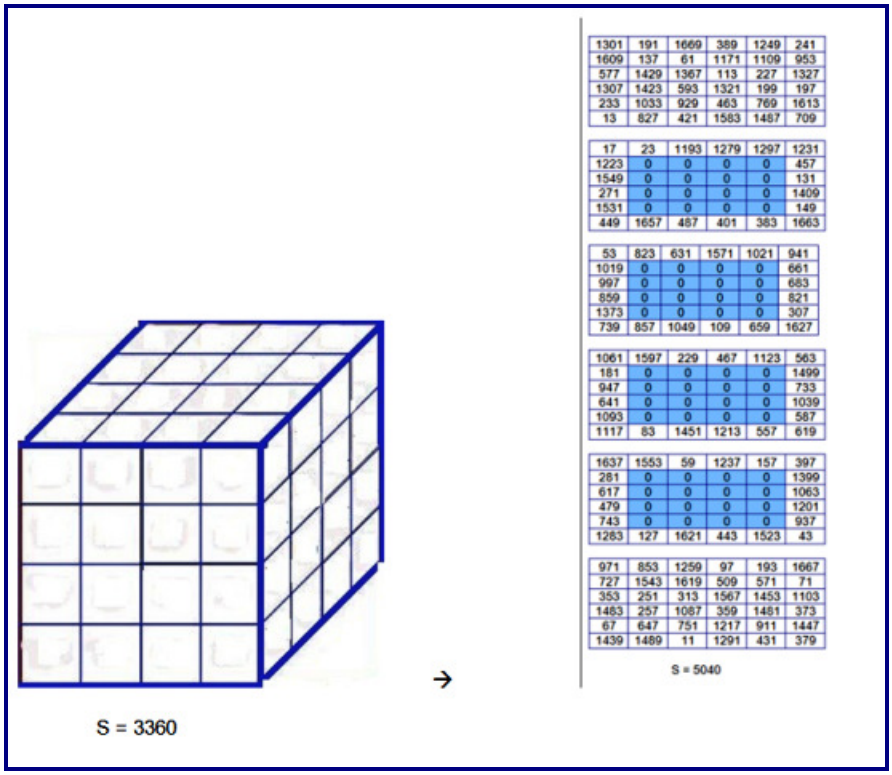
8237	2027	7283	5399	761	1493
3797	6791	2039	673	7297	4603
2473	659	6389	8263	1489	5927
1439	5623	2593	7757	827	6961
2347	3727	5779	107	7187	6053
6907	6373	1117	3001	7639	163

6113	2543	5351	2203	5821	3169
2837	5701	1163	7039	2371	6089
4073	7481	3613	347	8089	1597
6947	463	6857	6653	967	3313
1091	6133	4987	4931	229	7829
4139	2879	3229	4027	7723	3203

I was unable to make a concentric magic cube of order 6 with a magic constant S = 5040, perhaps a reader will have better luck.

The border is easily made. One embodiment is shown in Fig. 16.

Figure 16



Try to find a magic cube of order 4 with a magic constant S = 3360, which is to be inserted into the fringes, shown on the right in the illustration. It's an interesting challenge!

My partial solution to the problem that contains 5 errors appears in figure 17.

Figure 17

1301	191	1669	389	1249	241
1609	137	61	1171	1109	953
577	1429	1367	113	227	1327
1307	1423	593	1321	199	197
233	1033	929	463	769	1613
13	827	421	1583	1487	709

17	23	1193	1279	1297	1231
1223	1987	7	863	503	457
1549	613	2221	107	419	131
271	239	101	2053	967	1409
1531	521	1031	337	1471	149
449	1657	487	401	383	1663

53	823	631	1571	1021	941
1019	103	73	1997	1187	661
997	1847	673	317	523	683
859	601	461	1009	1289	821
1373	809	2153	37	361	307
739	857	1049	109	659	1627

1061	1597	229	467	1123	563
181	701	2099	349	211	1499
947	409	311	919	1721	733
641	1973	787	269	331	1039
1093	277	163	1823	1097	587
1117	83	1451	1213	557	619

1637	1553	59	1237	157	397
281	569	1181	151	1459	1399
617	491	155	2017	697	1063
479	547	2011	29	773	1201
743	1753	13	1163	431	937
1283	127	1621	443	1523	43

971	853	1259	97	193	1667
727	1543	1619	509	571	71
353	251	313	1567	1453	1103
1483	257	1087	359	1481	373
67	647	751	1217	911	1447
1439	1489	11	1291	431	379

Explanation of the errors:

361 is not prime
155 is not prime
697 is not prime
13 is not unique
431 is not unique

Another embodiment of the border appears in figure 18.

Figure 18

1327	739	479	821	1663	11
1453	149	13	1367	1087	971
1307	1093	1637	647	127	229
181	1291	1283	769	257	1259
641	1301	307	1237	383	1171
131	467	1321	199	1523	1399

23	53	1567	1021	1429	947
883					797
857					823
1483					197
1061					619
733	1627	113	659	251	1657

191	109	1063	853	1223	1601
937					743
631					1049
1619					61
1583					97
79	1571	617	827	457	1489

1597	1439	449	463	431	661
571					1109
67					1613
1103					577
683					997
1019	241	1231	1217	1249	83

1621	1487	1123	401	137	271
487					1193
727					953
233					1447
563					1117
1409	193	557	1279	1543	59

281	1213	359	1481	157	1549
709	1531	1667	313	593	227
1451	587	43	1033	1553	373
421	389	397	911	1423	1499
509	379	1373	443	1297	1039
1669	941	1201	859	17	353

I found a solution for this variant that only has three errors and it appears in figure 19.

Explanation of the errors:

445 is not prime
781 is not prime
97 is not unique

Figure 19

S = 5040, k = 1680

1327	739	479	821	1663	11
1453	149	13	1367	1087	971
1307	1093	1637	647	127	229
181	1291	1283	769	257	1259
641	1301	307	1237	383	1171
131	467	1321	199	1523	1399

23	53	1567	1021	1429	947
883	1987	7	863	503	797
857	613	2221	107	419	823
1483	239	101	2053	967	197
1061	521	1031	337	1471	619
733	1627	113	659	251	1657

191	109	1063	853	1223	1601
937	103	73	1997	1187	743
631	1847	757	317	439	1049
1619	601	461	1009	1289	61
1583	809	2069	37	445	97
79	1571	617	827	457	1489

1597	1439	449	463	431	661
571	701	2099	349	211	1109
67	409	311	919	1721	1613
1103	1973	787	269	331	577
683	277	163	1823	1097	997
1019	241	1231	1217	1249	83

1621	1487	1123	401	137	271
487	569	1181	151	1459	1193
727	491	71	2017	781	953
233	547	2011	29	773	1447
563	1753	97	1163	347	1117
1409	193	557	1279	1543	59

281	1213	359	1481	157	1549
709	1531	1667	313	593	227
1451	587	43	1033	1553	373
421	389	397	911	1423	1499
509	379	1373	443	1297	1039
1669	941	1201	859	17	353

Figure 20 contains a classic concentric magic cube of order 6. This cube is made up of distinct positive integers from 1 to 216. The interior is an unconventional associative magic cube of order 4 with magic constant S = 434.

Figure 20

S = 651, k = 217.

107	160	56	78	133	117
104	165	51	76	134	121
193	155	70	81	37	115
9	153	87	146	179	77
188	17	214	181	16	35
50	1	173	89	152	186

75	48	82	42	194	210
156	2	99	144	189	61
162	79	46	197	112	55
148	150	183	8	93	69
103	203	106	85	40	114
7	169	135	175	23	142

108	12	45	192	174	120
74	98	27	164	145	143
129	47	54	137	196	88
127	158	151	92	33	90
116	131	202	41	60	101
97	205	172	25	43	109

123	30	211	206	63	18
95	157	176	15	86	122
26	184	125	66	59	191
159	21	80	163	170	58
49	72	53	190	119	168
199	187	6	11	154	94

207	185	213	5	22	19
126	177	132	111	14	91
39	124	209	34	67	178
68	105	20	171	138	149
13	28	73	118	215	204
198	32	4	212	195	10

31	216	44	128	65	167
96	52	166	141	83	113
102	62	147	136	180	24
140	64	130	71	38	208
182	200	3	36	201	29
100	57	161	139	84	110

Concentric magic cubes of order 7

Figure 21 contains a scheme for concentric magic cubes of order 7.

Figure 21

y1	y2	y3	y4	y5	y6	y7
y8	y9	y10	y11	y12	y13	y14
y15	y16	y17	y18	y19	y20	y21
y22	y23	y24	y25	y26	y27	y28
y29	y30	y31	y32	y33	y34	y35
y36	y37	y38	y39	y40	y41	y42
y43	y44	y45	y46	y47	y48	y49

x1	x2	x3	x4	x5	x6	x7
x8						k-x8
x9						k-x9
x10						k-x10
x11						k-x11
x12						k-x12
k-x7	k-x2	k-x3	k-x4	k-x5	k-x6	k-x1

x13	x14	x15	x16	x17	x18	x19
x20						k-x20
x21						k-x21
x22						k-x22
x23						k-x23
x24						k-x24
k-x19	k-x14	k-x15	k-x16	k-x17	k-x18	k-x13

x25	x26	x27	x28	x29	x30	x31
x32						k-x32
x33						k-x33
x34			k/2			k-x34
x35						k-x35
x36						k-x36
k-x31	k-x26	k-x27	k-x28	k-x29	k-x30	k-x25

x37	x38	x39	x40	x41	x42	x43
x44						k-x44
x45						k-x45
x46						k-x46
x47						k-x47
x48						k-x48
k-x43	k-x38	k-x39	k-x40	k-x41	k-x42	k-x37

x49	x50	x51	x52	x53	x54	x55
x56						k-x56
x57						k-x57
x58						k-x58
x59						k-x59
x60						k-x60
k-x55	k-x50	k-x51	k-x52	k-x53	k-x54	k-x49

k-y49	k-y44	k-y45	k-y46	k-y47	k-y48	k-y43
k-y14	k-y9	k-y10	k-y11	k-y12	k-y13	k-8
k-y21	k-y16	k-y17	k-y18	k-y19	k-y20	k-y15
k-y28	k-y23	k-y24	k-y25	k-y26	k-y27	k-y22
k-y35	k-y30	k-y31	k-y32	k-y33	k-y34	k-y29
k-y42	k-y37	k-y38	k-y39	k-y40	k-y41	k-y36
k-y7	k-y2	k-y3	k-y4	k-y5	k-y6	k-y1

The magic constant is S = 7k / 2.

In general, the interior can be any magic cube of order 5 with magic constants S = 5k / 2, but it can also be a concentric magic cube.

Figure 22 contains an example of a classical concentric magic cube where the inside is not a concentric magic cube of order 5. This cube is made up of distinct positive integers from 1 to 343.
The interior of the cube is an unconventional associative pantriagonal cube of order 5 with a magic constant S = 860.

Figure 22

S = 1204, k = 344

318	8	57	261	7	227	326
312	11	60	258	10	222	331
17	305	163	103	136	324	156
51	137	170	169	299	196	182
91	292	245	267	161	84	64
105	235	306	50	300	66	142
310	216	203	96	291	85	3

200	73	111	123	255	176	266
1	296	230	199	133	2	343
304	28	322	256	160	94	40
72	120	54	283	217	186	272
219	212	81	15	309	243	125
330	204	173	107	41	335	14
78	271	233	221	89	168	144

116	97	79	25	320	254	313
340	223	192	126	30	289	4
110	315	249	153	87	56	234
274	82	276	210	179	113	70
215	74	43	302	236	205	129
118	166	100	69	328	197	226
31	247	265	319	24	90	228

19	242	187	59	323	180	194
214	185	119	23	282	251	130
195	277	146	80	49	308	149
193	269	238	172	106	75	151
232	36	295	264	198	67	112
201	93	62	321	225	159	143
150	102	157	285	21	164	325

33	339	297	268	131	71	65
135	147	16	275	244	178	209
298	139	108	42	301	270	46
206	231	165	134	68	262	138
63	288	257	191	95	29	281
190	55	314	218	152	121	154
279	5	47	76	213	273	311

177	317	332	220	115	37	6
189	9	303	237	171	140	155
92	101	35	329	263	132	252
246	158	127	61	290	224	98
104	250	184	88	22	316	240
58	342	211	145	114	48	286
338	27	12	124	229	307	167

341	128	141	248	53	259	34
13	333	284	86	334	122	32
188	39	181	241	208	20	327
162	207	174	175	45	148	293
280	52	99	77	183	260	253
202	109	38	294	44	278	239
18	336	287	83	337	117	26

Next, we will consider the case where a concentric magic cube of order 5 is inside a concentric magic cube of order 7.

The scheme of this magic cube is shown in Fig. 23.

Figure 23

y1	y2	y3	y4	y5	y6	y7
y8	y9	y10	y11	y12	y13	y14
y15	y16	y17	y18	y19	y20	y21
y22	y23	y24	y25	y26	y27	y28
y29	y30	y31	y32	y33	y34	y35
y36	y37	y38	y39	y40	y41	y42
y43	y44	y45	y46	y47	y48	y49

x1	x2	x3	x4	x5	x6	x7
x8	z1	z2	z3	z4	z5	k-x8
x9	z6	z7	z8	z9	z10	k-x9
x10	z11	z12	z13	z14	z15	k-x10
x11	z16	z17	z18	z19	z20	k-x11
x12	z21	z22	z23	z24	z25	k-x12
k-x7	k-x2	k-x3	k-x4	k-x5	k-x6	k-x1

x13	x14	x15	x16	x17	x18	x19
x20	z26	z27	z28	z29	z30	k-x20
x21	z31				k-z31	k-x21
x22	z32				k-z32	k-x22
x23	z33				k-z33	k-x23
x24	k-z30	k-z27	k-z28	k-z29	k-z26	k-x24
k-x19	k-x14	k-x15	k-x16	k-x17	k-x18	k-x13

x25	x26	x27	x28	x29	x30	x31
x32	z34	z35	z36	z37	z38	k-x32
x33	z39				k-z39	k-x33
x34	z40		k/2		k-z40	k-x34
x35	z41				k-z41	k-x35
x36	k-z38	k-z35	k-z36	k-z37	k-z34	k-x36
k-x31	k-x26	k-x27	k-x28	k-x29	k-x30	k-x25

x37	x38	x39	x40	x41	x42	x43
x44	z42	z43	z44	z45	z46	k-x44
x45	z47				k-z47	k-x45
x46	z48				k-z48	k-x46
x47	z49				k-z49	k-x47
x48	k-z46	k-z43	k-z44	k-z45	k-z42	k-x48
k-x43	k-x38	k-x39	k-x40	k-x41	k-x42	k-x37

x49	x50	x51	x52	x53	x54	x55
x56	k-z25	k-z22	k-z23	k-z24	k-z21	k-x56
x57	k-z10	k-z7	k-z8	k-z9	k-z6	k-x57
x58	k-z15	k-z12	k-z13	k-z14	k-z11	k-x58
x59	k-z20	k-z17	k-z18	k-z19	k-z16	k-x59
x60	k-z5	k-z2	k-z3	k-z4	k-z1	k-x60
k-x55	k-x50	k-x51	k-x52	k-x53	k-x54	k-x49

k-y49	k-y44	k-y45	k-y46	k-y47	k-y48	k-y43
k-y14	k-y9	k-y10	k-y11	k-y12	k-y13	k-8
k-y21	k-y16	k-y17	k-y18	k-y19	k-y20	k-y15
k-y28	k-y23	k-y24	k-y25	k-y26	k-y27	k-y22
k-y35	k-y30	k-y31	k-y32	k-y33	k-y34	k-y29
k-y42	k-y37	k-y38	k-y39	k-y40	k-y41	k-y36
k-y7	k-y2	k-y3	k-y4	k-y5	k-y6	k-y1

The magic constant of this concentric cube is S = 7k / 2.

Inside the concentric cube of order 7 is a concentric cube of order 5 with a magic constant S = 5k / 2. Furthermore, inside the concentric cube of order 5 is a magic cube of order 3 with a magic constant S = 3k / 2.

Our first step is to show the classic concentric magic cube of order 7 of this type, and it appears in figure 24.

Figure 24

S = 1204, k = 344

17	206	28	315	41	325	272
58	295	139	87	231	194	200
191	328	143	228	79	137	98
190	147	326	232	77	135	97
189	133	141	234	323	75	109
238	51	275	70	174	221	175
321	44	152	38	279	117	253

212	73	94	101	309	125	290
131	26	102	230	229	273	213
168	119	259	254	104	124	176
261	262	118	103	255	122	83
249	140	145	146	151	278	95
129	313	236	127	121	63	215
54	271	250	243	35	219	132

99	84	78	46	299	276	322
301	40	3	142	337	338	43
287	148	182	333	1	196	57
42	332	310	20	186	12	302
159	334	24	163	329	10	185
294	6	341	202	7	304	50
22	260	266	298	45	68	245

156	76	244	105	297	165	161
285	173	308	107	62	210	59
86	216	319	2	195	128	258
284	30	48	172	296	314	60
74	307	149	342	25	37	270
136	134	36	237	282	171	208
183	268	100	239	47	179	188

312	160	280	251	67	53	81
52	340	339	164	9	8	292
130	157	15	181	320	187	214
92	14	158	324	34	330	252
178	13	343	11	162	331	166
177	336	5	180	335	4	167
263	184	64	93	277	291	32

317	305	288	80	126	33	55
233	281	108	217	223	31	111
96	220	85	90	240	225	248
88	222	226	241	89	82	256
120	66	199	198	193	204	224
61	71	242	114	115	318	283
289	39	56	264	218	311	27

91	300	192	306	65	227	23
144	49	205	257	113	150	286
246	16	201	116	265	207	153
247	197	18	112	267	209	154
235	211	203	110	21	269	155
169	293	69	274	170	123	106

A concentric magic cube of order 7 of distinct primes appears in figure 25.

This magic cube is not easy to make. Inside is a concentric magic cube of order 5 with a magic constant S = 54515. Inside the cube of order 5 is a magic cube of order 3 with a magic constant S = 32709.

Figure 25

S = 76321, k = 21806

1783	19429	2593	3373	14983	19423	14737
1399	13417	17203	17047	6883	19	20353
13267	14629	15727	859	4567	13339	13933
20347	3943	19249	1483	19777	4903	6619
787	11353	17383	17293	1033	10303	18169
20809	4507	1693	17713	11119	19267	1213
17929	9043	2473	18553	17959	9067	1297

9103	3583	5107	5737	18097	18757	15937
823	14347	21163	9349	4099	5557	20983
16993	3793	9883	20899	9463	10477	4813
15259	6379	15739	12919	10369	9109	6547
18307	13627	3673	3019	21397	12799	3499
9967	16369	4057	8329	9187	16573	11839
5869	18223	16699	16069	3709	3049	12703

6247	4363	4297	21319	20743	1993	17359
12073	4639	7	6607	21523	21739	9733
13879	7723	9769	21613	1327	14083	7927
8839	21727	20107	229	12373	79	12967
19753	20359	2833	10867	19009	1447	2053
11083	67	21799	15199	283	17167	10723
4447	17443	17509	487	1063	19813	15559

919	5743	14107	18199	13597	9319	14437
19717	8803	523	18919	12757	13513	2089
17209	13177	20143	157	12409	8629	4597
9973	13093	3169	10903	18637	8713	11833
16057	11149	9397	21649	1663	10657	5749
5077	8293	21283	2887	9049	13003	16729
7369	16063	7699	3607	8209	12487	20887

16693	17257	15607	13873	3967	8647	277
20929	21493	15073	6163	3517	8269	877
307	18493	2797	10939	18973	3313	21499
1987	619	9433	21577	1699	21187	19819
12157	373	20479	193	12037	21433	9649
2719	13537	6733	15643	18289	313	19087
21529	4549	6199	7933	17839	13159	5113

21067	13183	15277	10567	1087	5443	9697
19927	5233	17749	13477	12619	5437	1879
6793	11329	11923	907	12343	18013	15013
4729	12697	6067	8887	11437	15427	17077
5623	9007	18133	18787	409	8179	16183
6073	16249	643	12457	17707	7459	15733
12109	8623	6529	11239	20719	16363	739

20509	12763	19333	3253	3847	12739	3877
1453	8389	4603	4759	14923	21787	20407
7873	7177	6079	20947	17239	8467	8539
15187	17863	2557	20323	2029	16903	1459
3637	10453	4423	4513	20773	11503	21019
20593	17299	20113	4093	10687	2539	997
7069	2377	19213	18433	6823	2383	20023

This concentric magic cube is shown in the picture "Russian nesting dolls" (see Fig. 26).

Figure 26

An attempt was made to make a concentric magic cube of order 7 of distinct primes with magic constant S < 76321. My attempts were not successful. I show two partial solutions (Fig. 27 and Fig. 28).

A challenge for the readers - find the edging! There is no guarantee that the problem has a solution.

Figure 27

S = 69769, k = 19934

y1	y2	y3	y4	y5	y6	y7
y8	y9	y10	y11	y12	y13	y14
y15	y16	y17	y18	y19	y20	y21
y22	y23	y24	y25	y26	y27	y28
y29	y30	y31	y32	y33	y34	y35
y36	y37	y38	y39	y40	y41	y42
y43	y44	y45	y46	y47	y48	y49

x1	x2	x3	x4	x5	x6	x7
x8	2857	7237	17683	5641	16417	k-x8
x9	8761	4861	5827	15733	14653	k-x9
x10	7333	13063	15271	14011	157	k-x10
x11	15607	16747	4027	1483	11971	k-x11
x12	15277	7927	7027	12967	6637	k-x12
k-x7	k-x2	k-x3	k-x4	k-x5	k-x6	k-x1

x13	x14	x15	x16	x17	x18	x19
x20	2143	7	8431	19387	19867	k-x20
x21	10333	12541	17317	43	9601	k-x21
x22	18211	14767	4483	10651	1723	k-x22
x23	19081	2593	8101	19207	853	k-x23
x24	67	19927	11503	547	17791	k-x24
k-x19	k-x14	k-x15	k-x16	k-x17	k-x18	k-x13

x25	x26	x27	x28	x29	x30	x31
x32	11941	10831	8803	17377	883	k-x32
x33	14947	16633	751	12517	4987	k-x33
x34	73	5851	9967	14083	19861	k-x34
x35	3823	7417	19183	3301	16111	k-x35
x36	19051	9103	11131	2557	7993	k-x36
k-x31	k-x26	k-x27	k-x28	k-x29	k-x30	k-x25

x37	x38	x39	x40	x41	x42	x43
x44	19597	19753	2011	463	8011	k-x44
x45	10513	727	11833	17341	9421	k-x45
x46	4441	9283	15451	5167	15493	k-x46
x47	3361	19891	2617	7393	16573	k-x47
x48	11923	181	17923	19471	337	k-x48
k-x43	k-x38	k-x39	k-x40	k-x41	k-x42	k-x37

x49	x50	x51	x52	x53	x54	x55
x56	13297	12007	12907	6967	4657	k-x56
x57	5281	15073	14107	4201	11173	k-x57
x58	19777	6871	4663	5923	12601	k-x58
x59	7963	3187	15907	18451	4327	k-x59
x60	3517	12697	2251	14293	17077	k-x60
k-x55	k-x50	k-x51	k-x52	k-x53	k-x54	k-x49

k-y49	k-y44	k-y45	k-y46	k-y47	k-y48	k-y43
k-y14	k-y9	k-y10	k-y11	k-y12	k-y13	k-8
k-y21	k-y16	k-y17	k-y18	k-y19	k-y20	k-y15
k-y28	k-y23	k-y24	k-y25	k-y26	k-y27	k-y22
k-y35	k-y30	k-y31	k-y32	k-y33	k-y34	k-y29
k-y42	k-y37	k-y38	k-y39	k-y40	k-y41	k-y36
k-y7	k-y2	k-y3	k-y4	k-y5	k-y6	k-y1

Figure 28

S = 68999, k = 19714

y1	y2	y3	y4	y5	y6	y7
y8	y9	y10	y11	y12	y13	y14
y15	y16	y17	y18	y19	y20	y21
y22	y23	y24	y25	y26	y27	y28
y29	y30	y31	y32	y33	y34	y35
y36	y37	y38	y39	y40	y41	y42
y43	y44	y45	y46	y47	y48	y49

x1	x2	x3	x4	x5	x6	x7
x8	16253	2141	14747	8501	7643	k-x8
x9	2087	18617	14771	8093	5717	k-x9
x10	1877	11831	4073	19463	12041	k-x10
x11	17891	3083	2243	12671	13397	k-x11
x12	11177	13613	13451	557	10487	k-x12
k-x7	k-x2	k-x3	k-x4	k-x5	k-x6	k-x1

x13	x14	x15	x16	x17	x18	x19
x20	1667	1637	7001	19403	19577	k-x20
x21	8627	7817	18731	3023	11087	k-x21
x22	19421	17417	1373	10781	293	k-x22
x23	19433	4337	9467	15767	281	k-x23
x24	137	18077	12713	311	18047	k-x24
k-x19	k-x14	k-x15	k-x16	k-x17	k-x18	k-x13

x25	x26	x27	x28	x29	x30	x31
x32	2927	19697	12911	1523	12227	k-x32
x33	16871	17807	593	11171	2843	k-x33
x34	17783	3221	9857	16493	1931	k-x34
x35	4217	8543	19121	1907	15497	k-x35
x36	7487	17	6803	18191	16787	k-x36
k-x31	k-x26	k-x27	k-x28	k-x29	k-x30	k-x25

x37	x38	x39	x40	x41	x42	x43
x44	19211	19709	8363	701	1301	k-x44
x45	7703	3947	10247	15377	12011	k-x45
x46	2531	8933	18341	2297	17183	k-x46
x47	1427	16691	983	11897	18287	k-x47
x48	18413	5	11351	19013	503	k-x48
k-x43	k-x38	k-x39	k-x40	k-x41	k-x42	k-x37

x49	x50	x51	x52	x53	x54	x55
x56	9227	6101	6263	19157	8537	k-x56
x57	13997	1097	4943	11621	17627	k-x57
x58	7673	7883	15641	251	17837	k-x58
x59	6317	16631	17471	7043	1823	k-x59
x60	12071	17573	4967	11213	3461	k-x60
k-x55	k-x50	k-x51	k-x52	k-x53	k-x54	k-x49

k-y49	k-y44	k-y45	k-y46	k-y47	k-y48	k-y43
k-y14	k-y9	k-y10	k-y11	k-y12	k-y13	k-8
k-y21	k-y16	k-y17	k-y18	k-y19	k-y20	k-y15
k-y28	k-y23	k-y24	k-y25	k-y26	k-y27	k-y22
k-y35	k-y30	k-y31	k-y32	k-y33	k-y34	k-y29
k-y42	k-y37	k-y38	k-y39	k-y40	k-y41	k-y36
k-y7	k-y2	k-y3	k-y4	k-y5	k-y6	k-y1

Concentric magic cubes of order 8

On the web page

http://www.magic-SquareS.net/c-t-htm/c_prime.htm

you will find the concentric magic cube of order 8 of distinct primes that appears in figure 29.

The magic constant of the cube is S = 39480. Inside is a concentric magic cube of order 6 with a magic constant S = 29610, this is the magic cube that appeared in figure 1. Inside the concentric cube of order 6 is an associative and pantriagonal cube of order 4 with a magic constant S = 19740.

Figure 29

S = 39480, k = 9870

13	9859	6679	9829	2129	53	6869	4049
1637	9781	103	8171	181	7577	9733	2297
9511	349	3623	269	433	9787	7691	7817
9631	257	7331	2477	9371	9413	521	479
9283	1039	941	631	8837	661	8861	9227
6803	709	3613	8443	9187	3541	2617	4567
1493	8707	9043	907	8291	6701	1171	3167
1109	8779	8147	8753	1051	1747	2017	7877

811	9127	7841	5867	7211	2909	3931	1783
6781	4831	4783	67	9811	4639	5479	3089
4229	191	241	193	9473	9769	9743	5641
409	331	577	5009	4751	9619	9323	9461
7177	8273	9719	8933	1123	829	733	2693
4967	8423	7499	8287	1789	1801	1811	4903
7019	7561	6791	7121	2663	2953	2521	2851
8087	743	2029	4003	2659	6961	5939	9059

8431	1289	4951	4933	1063	8941	9013	859
5717	131	761	379	9403	9497	9439	4153
6151	8951	2437	3547	5309	8447	919	3719
19	9643	3209	5573	2281	8677	227	9851
2711	2143	8243	4877	6007	613	7727	7159
3307	8311	5851	5743	6143	2003	1559	6563
4133	431	9109	9491	467	373	9739	5737
9011	8581	4919	4937	8807	929	857	1439

8783	1181	8093	1759	1933	6379	2633	8719
4201	337	8849	8821	1409	1307	8887	5669
3671	7013	5903	2879	9007	1951	2857	6199
8641	8009	3217	2767	8117	5639	1861	1229
7523	9049	6073	5521	2333	5813	821	2347
2719	4219	4547	8573	283	6337	5651	7151
2791	983	1021	1049	8461	8563	9533	7079
1151	8689	1777	8111	7937	3491	7237	1087

8669	1223	1483	7583	2267	7477	5197	5581
3779	8543	8839	9277	173	1831	947	6091
3917	4177	3533	9587	1297	5323	5693	5953
5881	7487	4057	7537	4349	3797	2383	3989
2381	31	4231	1753	7103	6653	9839	7489
3803	449	7919	863	6991	3967	9421	6067
6761	8923	1031	593	9697	8039	1327	3109
4289	8647	8387	2287	7603	2393	4673	1201

7529	7993	2111	3041	7789	3889	3947	3181
3449	8419	3299	8317	1607	5419	2549	6421
4021	9151	7867	3727	4127	4019	719	5849
1481	3593	9257	3863	4993	1627	6277	8389
2113	977	1193	7589	4297	6661	8893	7757
7129	149	1423	4561	6323	7433	9721	2741
7069	7321	6571	1553	8263	4451	1451	2801
6689	1877	7759	6829	2081	5981	5923	2341

3251	7717	6599	5351	8269	1709	37	6547
6343	7349	3079	2749	7207	6917	2309	3527
5927	127	9629	9677	397	101	9679	3943
4027	547	9293	4861	5119	251	9539	5843
7649	9137	151	937	8747	9041	1597	2221
5449	8059	2371	1583	8081	8069	1447	4421
3511	4391	5087	9803	59	5231	5039	6359
3323	2153	3271	4519	1601	8161	9833	6619

1993	1091	1723	1117	8819	8123	7853	8761
7573	89	9767	1699	9689	2293	137	8233
2053	9521	6247	9601	9437	83	2179	359
9391	9613	2539	7393	499	457	9349	239
643	8831	8929	9239	1033	9209	1009	587
5303	9161	6257	1427	683	6329	7253	3067
6703	1163	827	8963	1579	3169	8699	8377
5821	11	3191	41	7741	9817	3001	9857

The scheme for the concentric magic cube of order 8, based on the standard cube depicted in Fig. 29, appears in figure 30.

Figure 30

S = 4k

x1	x2	x3	x4	x5	x6	x7	x8
x9	x10	x11	x12	x13	x14	x15	x16
x17	x18	x19	x20	x21	x22	x23	x24
x25	x26	x27	x28	x29	x30	x31	x32
x33	x34	x35	x36	x37	x38	x39	x40
x41	x42	x43	x44	x45	x46	x47	x48
x49	x50	x51	x52	x53	x54	x55	x56
x57	x58	x59	x60	x61	x62	x63	x64

x65	x66	x67	x68	x69	x70	x71	x72
x73	z1	z2	z3	z4	z5	z6	k-x73
x74	z7	z8	z9	z10	z11	z12	k-x74
x75	z13	z14	z15	z16	z17	z18	k-x75
x76	z19	z20	z21	z22	z23	z24	k-x76
x77	z25	z26	z27	z28	z29	z30	k-x77
x78	z31	z32	z33	z34	z35	z36	k-x78
k-x72	k-x66	k-x67	k-x68	k-x69	k-x70	k-x71	k-x65

x79	x80	x81	x82	x83	x84	x85	x86
x87	z37	z38	z39	z40	z41	z42	k-x87
x88	z43					k-z43	k-x88
x89	z44					k-z44	k-x89
x90	z45					k-z45	k-x90
x91	z46					k-z46	k-x91
x92	k-z42	k-z38	k-z39	k-z40	k-z41	k-z37	k-x92
k-x86	k-x80	k-x81	k-x82	k-x83	k-x84	k-x85	k-x79

x93	x94	x95	x96	x97	x98	x99	x100
x101	z47	z48	z49	z50	z51	z52	k-x101
x102	z53					k-z53	k-x102
x103	z54					k-z54	k-x103
x104	z55					k-z55	k-x104
x105	z56					k-z56	k-x105
x106	k-z52	k-z48	k-z49	k-z50	k-z51	k-z47	k-x106
k-x100	k-x94	k-x95	k-x96	k-x97	k-x98	k-x99	k-x93

x107	x108	x109	x110	x111	x112	x113	x114
x115	z57	z58	z59	z60	z61	z62	k-x115
x116	z63					k-z63	k-x116
x117	z64					k-z64	k-x117
x118	z65					k-z65	k-x118
x119	z66					k-z66	k-x119
x120	k-z62	k-z58	k-z59	k-z60	k-z61	k-z57	k-x120
k-x114	k-x108	k-x109	k-x110	k-x111	k-x112	k-x113	k-x107

x121	x122	x123	x124	x125	x126	x127	x128
x129	z67	z68	z69	z70	z71	z72	k-x129
x130	z73					k-z73	k-x130
x131	z74					k-z74	k-x131
x132	z75					k-z75	k-x132
x133	z76					k-z76	k-x133
x134	k-z72	k-z68	k-z69	k-z70	k-z71	k-z67	k-x134
k-x128	k-x122	k-x123	k-x124	k-x125	k-x126	k-x127	k-x121

x135	x136	x137	x138	x139	x140	x141	x142
x143	k-z36	k-z32	k-z33	k-z34	k-z35	k-z31	k-x143
x144	k-z12	k-z8	k-z9	k-z10	k-z11	k-z7	k-x144
x145	k-z18	k-z14	k-z15	k-z16	k-z17	k-z13	k-x145
x146	k-z24	k-z20	k-z21	k-z22	k-z23	k-z19	k-x146
x147	k-z30	k-z26	k-z27	k-z28	k-z29	k-z25	k-x147
x148	k-z6	k-z2	k-z3	k-z4	k-z5	k-z1	k-x148
k-x142	k-x136	k-x137	k-x138	k-x139	k-x140	k-x141	k-x135

k-x64	k-x58	k-x59	k-x60	k-x61	k-x62	k-x63	k-x57
k-x16	k-x10	k-x11	k-x12	k-x13	k-x14	k-x15	k-x9
k-x24	k-x18	k-x19	k-x20	k-x21	k-x22	k-x23	k-x17
k-x32	k-x26	k-x27	k-x28	k-x29	k-x30	k-x31	k-x25
k-x40	k-x34	k-x35	k-x36	k-x37	k-x38	k-x39	k-x33
k-x48	k-x42	k-x43	k-x44	k-x45	k-x46	k-x47	k-x41
k-x56	k-x50	k-x51	k-x52	k-x53	k-x54	k-x55	k-x49
k-x8	k-x2	k-x3	k-x4	k-x5	k-x6	k-x7	k-x1

An attempt was made to make this scheme a concentric magic cube of order 8 of distinct primes. The best result was the incomplete solution that appears in figure 31.

Figure 31

S = 33600, k = 8400

4817	2819	2269	7517	3067	4871	4783	3457
4889	2741	2137	7523	2963	4909	4729	3709
6067	2557	5009	5281	1373	7541	839	4933
7019	1619	7349	1249	4289	1931	7247	2897
5849	3319	8093	3833	5743	3253	1709	1801
1951	6829	647	4273	3919	2617	5647	7717
2731	5483	5557	3167	5039	797	6949	3877
277	8233	2539	757	7207	7681	1697	5209

x65	x66	x67	x68	x69	x70	x71	x72
x73	5197	5521	5171	4373	677	4261	k-x73
x74	2311	2699	7237	1361	6029	5563	k-x74
x75	6803	919	4787	8053	311	4327	k-x75
x76	5087	7937	1543	1747	7433	1453	k-x76
x77	571	2267	3413	3469	8171	7309	k-x77
x78	5231	5857	3049	6197	2579	2287	k-x78
k-x72	k-x66	k-x67	k-x68	k-x69	k-x70	k-x71	k-x65

x79	x80	x81	x82	x83	x84	x85	x86
x87	523	809	11	8269	8011	7577	k-x87
x88	37	1213	8293	2621	4673	8363	k-x88
x89	8161	7573	643	5807	2777	239	k-x89
x90	8287	6911	137	2011	7741	113	k-x90
x91	7369	1103	7727	6361	1609	1031	k-x91
x92	823	7591	8389	131	389	7877	k-x92
k-x86	k-x80	k-x81	k-x82	k-x83	k-x84	k-x85	k-x79

x93	x94	x95	x96	x97	x98	x99	x100
x101	179	6763	5717	3389	5689	3463	k-x101
x102	8317	709	6397	6287	3407	83	k-x102
x103	2503	8387	1399	2153	4861	5897	k-x103
x104	2143	727	7823	31	8219	6257	k-x104
x105	7121	6977	1181	8329	313	1279	k-x105
x106	4937	1637	2683	5011	2711	8221	k-x106
k-x100	k-x94	k-x95	k-x96	k-x97	k-x98	k-x99	k-x93

x107	x108	x109	x110	x111	x112	x113	x114
x115	4951	7537	1667	1567	4241	5237	k-x115
x116	7901	8087	71	7219	1423	499	k-x116
x117	1187	181	8369	577	7673	7213	k-x117
x118	1297	3539	6247	7001	13	7103	k-x118
x119	6701	4993	2113	2003	7691	1699	k-x119
x120	3163	863	6733	6833	4159	3449	k-x120
k-x114	k-x108	k-x109	k-x110	k-x111	k-x112	k-x113	k-x107

x121	x122	x123	x124	x125	x126	x127	x128
x129	8237	2027	7283	5399	761	1493	k-x129
x130	3797	6791	2039	673	7297	4603	k-x130
x131	2473	659	6389	8263	1489	5927	k-x131
x132	1439	5623	2593	7757	827	6961	k-x132
x133	2347	3727	5779	107	7187	6053	k-x133
x134	6907	6373	1117	3001	7639	163	k-x134
k-x128	k-x122	k-x123	k-x124	k-x125	k-x126	k-x127	k-x121

x135	x136	x137	x138	x139	x140	x141	x142
x143	6113	2543	5351	2203	5821	3169	k-x143
x144	2837	5701	1163	7039	2371	6089	k-x144
x145	4073	7481	3613	347	8089	1597	k-x145
x146	6947	463	6857	6653	967	3313	k-x146
x147	1091	6133	4987	4931	229	7829	k-x147
x148	4139	2879	3229	4027	7723	3203	k-x148
k-x142	k-x136	k-x137	k-x138	k-x139	k-x140	k-x141	k-x135

3191	167	5861	7643	1193	719	6703	8123
4691	5659	6263	877	5437	3491	3671	3511
3467	5843	3391	3119	7027	859	7561	2333
5503	6781	1051	7151	4111	6469	1153	1381
6599	5081	307	4567	2657	5147	6691	2551
683	1571	7753	4127	4481	5783	2753	6449
4523	2917	2843	5233	3361	7603	1451	5669

A challenge is issued to the reader, find a complete solution.

Dear readers!
Should you find solutions to the proposed problems, please send them to the site

http://primesmagicgames.altervista.org/wp/competitions/

Furthermore, if you find a better solution, please send it in.

Links

1. http://en.wikipedia.org/wiki/Magic_cube
2. http://www.magic-SquareS.net/c-t-htm/c_prime.htm
3. http://primesmagicgames.altervista.org/wp/
4. Full size picture Fig. 26:
http://s017.radikal.ru/i427/1410/01/9dac57fb20e9.jpg

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Примечание: авторский перевод статьи на английский язык редактировал Charles Ashbacher. Он же написал аннотацию.